Algebra II Standards and Learning Activities

NUMBER SENSE AND OPERATIONS INDICATORS

AII.N.1. Know and use the properties of operations on real numbers, including the existence of the identity and inverse elements for addition and multiplication and the existence of *nth* roots of positive real numbers for any positive integer *n*, and the *nth* power of positive real numbers.

AII.N.2. Simplify numerical expressions with powers and roots, including fractional and negative exponents.

Example: Simplify the following expression: $\left(2a^{-2}b^3\right)^4 \left(4a^3 b^{-1}\right)^{-2}$

Example: Simplify the following expression: $\dfrac{2 \times 3^2 + 5 \times 3 \times 2^3}{30}$

AII.N.3. Know the representation of complex numbers (e.g., a + b*i* where a and b are real numbers) and the procedures for adding, multiplying, and inverting complex numbers. Understand the associative, commutative, and identity properties for complex arithmetic.

*Example: Multiply 7 – 4*i* by 10 + 6*i.

*Example: Plot the points 7 — 4*i* and 10 + 6*i* in the complex plane. Compute the sum of these two numbers and add the solution to your plot.*

PATTERNS, RELATIONS, AND ALGEBRA INDICATORS

AII.P.1. Describe, complete, extend, analyze, generalize, and create a wide variety of patterns, including iterative and recursive patterns such as Fibonacci Numbers and Pascal's Triangle.

Example: Find the middle term in the expanded form of the expression $\left(x - \dfrac{2}{x}\right)^{10}$

Example: The following shows the first 5 rows of Pascal's Triangle:

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            1
         1     1
      1     2     1
   1     3     3     1
1     4     6     4     1
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1. Construct the first 10 rows.

2. Identify different families or sets of numbers in the diagonals.

3. Relate the numbers in the triangle to the row numbers.

4. Examine sums of rows. Relate row sums to the row numbers.

5. For each row, form two sums by adding every other number. Compare sums within and between rows. Describe the patterns that emerge and why they occur.

6. Describe how the triangle is developed recursively.

AII.P.2. Identify arithmetic and geometric sequences and finite arithmetic and geometric series. Use the properties of such sequences and series to solve problems, including finding the formula for the general term and the sum, recursively and explicitly.

Example: Find the sum of the following (finite) series:

$$\frac{3}{5} + \frac{9}{25} + \frac{27}{125} + \dots + \frac{729}{15,625}$$

Example: Find the sum of the following (infinite) series:

$$\frac{3}{5} + \frac{9}{25} + \frac{27}{125} + \dots$$

Example: Find the 10th term in the following arithmetic series:

3, 7, 11, 15, ...

and compute the sum of these 10 terms.

(See also AII.P.1)

AII.P.3. Understand functional notation, evaluate a function at a specified point in its domain, and perform operations on functions with emphasis on the domain and range.

Example: If $f(x) = 7x + 2$ find the value of $f(3)$.

Example: If $f(x) = 7x + 2$, find W that gives $f(W) = 14$.

Example: If $f(x) = x^2$ and $g(x) = 3x - 1$, find and simplify each of the following:

(a) $\left(f \bullet g\right)(x)$;   (b) $\left(f + g\right)(x)$;   (c) $\left(g/f\right)(x)$;   (d) $f(g(x))$.

Example: If $f(x) = \dfrac{4}{x^2 + 8}$ then which of the following is equal to $f(2x)$?

(a) $\dfrac{8}{x^2 + 8}$;   (b) $\dfrac{4}{2x^2 + 8}$;   (c) $\dfrac{1}{x^2 + 2}$;   (d) $\dfrac{2}{x^2 + 4}$.

Example: Find the domain and range for the function

$$f(x) = 10 - \sqrt{81 - x^2}$$

AII.P.4. Understand exponential and logarithmic functions and their basic arithmetic properties, including change of base and formulas for exponential of a sum and logarithm of a product.

Example: Solve for x : $\log_3 (x + 1) - \log_3 (x) = 1$

Example: Simplify the following expressions:

(a) $\dfrac{\log_3 7}{\log_3 5}$

(b) $\log_{\sqrt{b}} b^2$

(c) $b^{\,3\log_b 2}$

Example: The pH for a solution is given by the following formula:

$$pH = \log \left(\dfrac{1}{H^+} \right)$$

where H^+ is the number of hydrogen ions per liter of the solution.

a) How many hydrogen ions are in each liter of a fruit juice with $H^+ = 3.9$?

b) What is the pH for a solution with $H^+ = 3.9$?

c) How does the pH of a solution change if the number of hydrogen ions per liter is increased by a factor of 10?

AII.P.5. Given algebraic, numeric, and/or graphical representations, recognize functions as polynomial, rational, logarithmic, or exponential, and describe their behavior.

Example: The three graphs shown at the right are best described as:

A) I – linear, II – quadratic, III-exponential
B) I – exponential, II – linear, III-quadratic
C) I – exponential, II – quadratic, III-linear
D) I – quadratic, II – exponential, III-linear
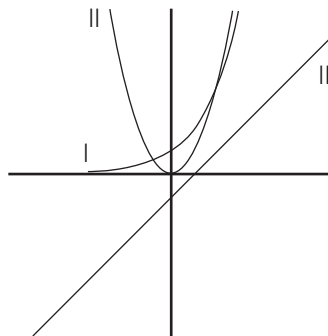
PATTERNS, RELATIONS, AND ALGEBRA INDICATORS (CONTINUED)

AII.P.6. Find solutions to radical equations; find solutions to quadratic equations (with real coefficients and real or complex roots) graphically, by factoring, by completing the square, or by using the quadratic formula.

Example: Solve $\sqrt{x-9} = 9 - \sqrt{x}$

(See also AII.P.3)

Example: Find all real solutions for $(x + 4) = \sqrt{x-2} - 2$

(See also AII.P.3)

Example: Find all points (if any exist) where the graph of $y = x^2 - 2$ *intersects the graph of* $y = x$.

(See also AII.P.9)

AII.P.7. *Solve a variety of equations and inequalities using algebraic, graphical, and numerical methods, including the quadratic formula. Include polynomial, exponential, and logarithmic functions, expressions involving the absolute values, and simple rational expressions.*

AII.P.8. *Explore matrices and their operations, including using them to solve systems of linear equations. Apply to solutions of everyday problems.*

Example: Rewrite the matrix equation as a system of equations and find the solution.

$$\begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 18 \end{bmatrix}$$

AII.P.9. Use symbolic, numeric, and graphical methods to solve systems of equations and/or inequalities involving algebraic, exponential, and logarithmic expressions. Describe the relationships among the methods.

AII.P.10. Solve everyday problems that can be modeled using polynomial, rational, exponential, logarithmic, and step functions; absolute values; and square roots. Apply appropriate graphical, tabular, or symbolic methods to the solution. Include compound interest, exponential growth and decay, and direct and inverse variation problems.

AII.P.11. Recognize translations and scale changes of a given function f(x) resulting from substitutions for the various parameters a, b, c, and d in $y = af(b(x + c/b)) + d$. In particular, describe qualitatively the effect of such changes on polynomial, rational, exponential, and logarithmic functions.

AII.P.12. Simplify rational expressions. Solve rational equations and inequalities.

Example: Simplify the expression $\dfrac{(x + h)^2 - x^2}{h}$ *as much as possible.*

Example: Find the zeros of $\dfrac{x^2 - 3x - 8}{2x^2 - 8}$

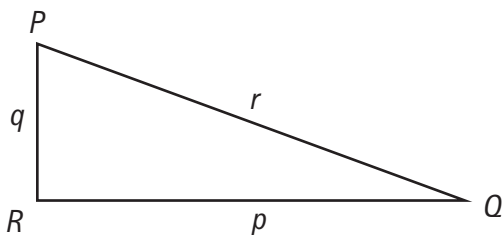*(See also AII.P.3, AII.P.7)*

GEOMETRY INDICATORS

AII.G.1. Define the sine, cosine, and tangent of an acute angle. Apply to the solution of problems.

Example: Triangle PQR is a scalene right triangle. Use the following figure to give an expression for each of the following:

sin*(P)*   cos*(P)*   tan*(P)*

Example: What can you say about a triangle if the lengths of the sides are 3, 4, and 6 inches?

Example: How far from the horizontal must a sheet of plywood 4 feet wide be rotated to fit through a doorway 30 inches wide?

AII.G.2. Explain the identity $\sin^2\theta + \cos^2\theta = 1$. Relate the identity to the Pythagorean theorem.

AII.G.3. Relate geometric and algebraic representations of lines and simple curves.

DATA ANALYSIS, STATISTICS, AND PROBABILITY INDICATORS

AII.D.1. Select an appropriate graphical representation for a set of data and use appropriate statistics (e.g., quartile or percentile distribution) to communicate information about the data, including box plots.

AII.D.2. Use combinatorics (e.g., fundamental counting principle, permutations, and combinations) to solve problems, including computing geometric probabilities and probabilities of compound events.

Example: How many ways can you choose 3 students from a group of 10 to serve on the math team?